On the triple pomeron contribution in the hard pomeron theory

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Abstract. Within the hard pomeron approach it is shown that the triple pomeron interaction gives no contribution to the cross-section for a projectile with large enough virtuality $g^2 \ln Q^2 \sim 1$. For small virtualities $g^2 \ln Q^2 << 1$ its contribution is essential.

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1 Introduction

Naive estimates seem to show that the triple pomeron diagram contributes significantly to the scattering cross-section at high energies. Take the diagram of Fig. 1a as an example. Let the total c.m. energy squared be $s = \exp Y$, those for the upper and lower pomerons $s_1 = \exp y_1$ and $s_2 = \exp y_2$, respectively, $y_1 + y_2 = Y$, and Δ the pomeron intercept. Then the contribution of the diagram seems to be

$$\gamma_1 \gamma_2 \gamma_{3P} \int_0^Y dy_1 \exp(\Delta y_1) \exp(2\Delta (Y - y_1)) = \gamma_1 \gamma_3 \gamma_{3P} \Delta^{-1} \exp(2\Delta Y) (1 - \exp(-\Delta Y))$$
 (1)

where γ_1 and γ_2 are the couplings of the colliding particle (the same for the projectile and target, for simplicity) to one and two pomerons and γ_{3P} is the triple pomeron coupling. This contribution is of the same order as the one from the "pure" two pomeron exchange, corresponding to the diagram of Fig. 1b

$$\gamma_2^2 \exp(2\Delta Y) \tag{2}$$

even in the small coupling limit, when one should take into account that $\gamma_{3P} \sim \gamma_2$.

However inspecting the result (1) one notices that the whole contribution to the righthand side at large Y comes from the region of integration on the left $y_1 \ll Y$, where the existence of the upper pomeron becomes questionable. So the conclusion that at large energies the triple pomeron gives a sizable contribution is not at all obvious. In fact, in our earlier paper [1] we assumed that it does not contribute, which lead us to the cross-section essentially of the eikonal form.

In this note we try to study the triple pomeron contribution in the framework of the BFKL hard pomeron [2] in some detail. We shall study only the simplest diagram of Fig. 1a and even for this diagram we shall not be able to obtain results in a form, amenable for numerical calculations. However, what we shall find is sufficient to draw definite conclusions as to the relative importance of the triple pomeron contribution. Since the derivation is very technical, we present here the main result together with its simple explanation.

As it turns out, the relative importance of the triple pomeron contribution depends on the magnitude of the colliding particle virtuality (or mass) Q^2 . If it is high enough (e.g. for the structure function) then the contribution of the triple pomeron is neglegible. However, with Q^2 diminishing, the triple pomeron contribution becomes more important and at low Q^2 it results at least of the same magnitude as the pure two pomeron exchange (Fig. 1b).

To understand this result note that the hard pomeron theory with a small (fixed) coupling constant g possesses an intrinsic large rapidity (or energy) scale

$$Y_0 = 1/g^2 \tag{3}$$

This should be contrasted to the "external" rapidity Y and colliding particle virtuality $Q^2 = \ln \zeta$. Evidently the pomeron theory applies only when

$$1 << Y_0 << Y, \quad \zeta << Y \tag{4}$$

However the relation between ζ and Y_0 may be arbitrary.

Now take the generic diagram of Fig. 2 corresponding to both of the diagrams in Fig. 1, with the upper blob B taken at the energy $s_1 = \exp y_1$. Its contribution is given by the integral similar to (1)

$$\gamma_2 \int_0^Y dy_1 B(y_1, \zeta) \exp(2\Delta(Y - y_1)) \tag{5}$$

The integration in (5) is evidently limited to values of y_1 smaller or of the order of $\Delta^{-1} \sim Y_0$. Then we may distinguish between two possibilities.

If Q^2 is large, so that ζ is of the order (or greater) than Y_0

$$\zeta \sim Y_0$$
 (6)

then blob B is, in fact, not in the low-x but in a pure perturbative regime (recall that the coupling constant is supposed to be fixed and small). Then, to the accuracy adopted in the BFKL theory, we have to retain only the lowest order contribution to it. This leads to the pure double pomeron exchange. For the photon projectile it means that the two pomerons should be directly coupled to the the $q\bar{q}$ loop. Thus in the region (6) the triple pomeron does not contribute: it simply does not exist.

On the other hand for small Q^2 , i.e. when

$$\zeta << Y_0 \tag{7}$$

blob B turns out to be in the low-x regime and we can take a pomeron for its leading contribution $B(y,\zeta) = \gamma_1 \gamma_{3P} P(g^2 y)$. The integral (5) then gives

$$\gamma_1 \gamma_2 \gamma_{3P} g^{-2} \exp(2\Delta Y) \int_0^{Y/g^2} dz P(z) \exp(-2z)$$
(8)

This contribution now does not reduce to the pure two-pomeron exchange and corresponds to the triple p[omeron of Fig. 1a. Note that in (8) finite values of z contribute, so that the upper pomeron of Fig. 1a enters here not in its asymptotic regime (as in (1)) but at finite values of $q^2 \ln s$ and so has to be taken in its full complexity.

As a result, we find that the hard pomeron theory allows for a clear distinction between colliding "particles", with $g^2 \ln Q^2 \ll 1$, and virtual probes, with $g^2 \ln Q^2 \sim 1$. For the former the triple pomeron dominates the cross-section. For the latter it does not contribute.

Turning to the literature, we recall that our results for summing multipomeron exchanges in [1] referred to the photon structure function at high Q^2 . So the omission of the triple (and

multiple) pomeron interactions, which was made there, results justified. On the other hand, in a series of papers of A.Mueller $et\ al.$ [3-5] the calculation of the double (and multiple) pomeron exchange seems to be made under the assumption of a fixed mass M of colliding "onia" and the coupling g going to zero. Then one inevitably arrives at the region (7) where the triple pomeron seems to give the bulk of the contribution. This may be a part of the explanation why the resulting amplitudes do not eikonalize in A.Mueller's approach.

We have to note, however, that to move closer to the realistic QCD, one has to take the onia mass M large enough to make the effective coupling constant small. This latter condition, translated into the fixed coupling constant language, means that $g^2 \ln M^2 \sim 1$, so that the region (6) should rather be considered, where no triple pomeron contribution is to be expected.

In the rest of this note we present the calculation of the triple pomeron contribution (Fig. 1a). As mentioned we shall not be able to find an expression for it which admits, say, numerical estimates. However we shall clearly see how and why this contribution goes to zero when Q^2 rises. Also a part (dominant at high Q^2) will be found in a form which can be studied numerically. Our calculations will be heavily based on the results obtained in our previous paper [6], where the triple pomeron vertex was studied in the asymptotic regime.

2 The triple pomeron contribution. The target part

The cross-section corresponding to Fig. 1a. in the hard pomeron approach of [2] is explicitly given by

$$\sigma = -\frac{g^{10}N^3}{32(2\pi)^9} \int d^2l \int (ds_1/s_1) \int \prod_{i=1}^3 (d^2r_i d^2r_i'(2\pi)^{-2}) K_l(r_1, r_2, r_3)$$

$$\exp(-il(r_2 + r_3)/2) G_0(s_1, r_1, r_1') G_l(s_2, r_2, r_2') G_{-l}(s_2, -r_3, r_3') \rho_1(r_1') \rho_2(r_2', r_3')$$
(9)

Here N is the number of flavours; $s_2 = s/s_1$ where s is the overall c.m. energy squared; r'_i and r_i , i = 1, 2, 3, are the initial and final transverse dimensions of the upper (i = 1) and lower (i = 2, 3) pomerons as they propagate from the sources (colliding particles) to the triple pomeron junction. The sources colour densities are ρ_1 for the projectile and ρ_2 for the target. The BFKL Green function $G_l(s, r, r')$ describes propagation of the pomeron with the total momentum l and energy s. $K_l(r_1, r_2, r_3)$ is the triple pomeron interaction vertex. It originates from the Bartels' vertex for the transition of 2 reggeized gluons into 4 [7]. Its explicit form in the coordinate space was found in [6]. Taking into account that the Green functions G_l vanish if r = 0 or r' = 0, it can be reduced to

$$K_l(r_1, r_2, r_3) = -\frac{2}{(2\pi)^2} \frac{(r_2 r_3)}{r_2^2 r_3^2} \nabla_1^4 \delta^2(r_1 + r_2 + r_3)$$
(10)

The numerical coefficient in (9) combines factors $g^2N/2$ and $g^4N/2$ separated from the colour densities ρ_1 and ρ_2 , respectively, a factor g^4N from the triple pomeron vertex, a factor 1/2 for the identity of two lower pomerons and some numerical factors which relate the cross-section to the amplitude corresponding to the diagram of Fig. 1a.

For our purpose the explicit form of the target density $\rho_2(r'_2, r'_3)$ will be inessencial, except that it should be well behaved and restrict r'_2 and r'_3 to some average value r_{20} of the order of the target transverse dimension. To simplify, we then take $\rho_2(r'_2, r'_3) = \delta^2(r'_2 - r'_3)\delta(r'_2 - r_{20})/(2\pi r_{20})$, which means that we simply substitute in the two Green functions for the lower pomerons $r'_2 = r'_2 = r_{20}$ where the latter vector has length r_{20} and an arbitrary direction. After this substitution, and with the explicit form of K_l given by (10), the cross-section (9) transforms into

$$\sigma = -\frac{g^{10}N^3}{16(2\pi)^{13}} \int d^2l \int (ds_1/s_1) \int d^2r_1 d^2q \rho_1(r_1)$$

$$\chi_1(s_1, q + l/2, r_1)(\nabla_q \chi_2(s_2, l, q))^2$$
(11)

where

$$\chi_1(s, q, r_1) = \int d^2r \nabla^4 G_0(s, r, r_1) \exp iqr$$
(12)

and

$$\chi_2(s, l, q) = \int d^2r r^{-2} G_l(s, r, r_{20}) \exp iqr$$
(13)

As mentioned in the Introduction, the triple pomeron interaction which enters (11) has been studied in [6] in the asymptotic region $s_1 \sim s_2 \to \infty$. In our case, we rather have to integrate over all s_1 , so that the upper pomeron will stay far from its asymptotic regime and we shall have to take the exact form for its Green function. However the target part χ_2 enters (11) in its asymptotic region, so that we may borrow its form from [6], to which paper we refer the reader for more detail in this respect.

The leading contribution to the BFKL Green function at $l \neq 0$, which appears in (13), has the form [8]

$$G_l(s, r, r') = (1/4\pi^2) \int \frac{d\nu \nu^2}{(\nu^2 + 1/4)^2} s^{\omega(\nu)} E_l^{\nu}(r) E_l^{-\nu}(r')$$
(14)

where

$$\omega(\nu) = (g^2 N / 2\pi^2)(\psi(1) - \text{Re}\psi(1/2 + i\nu))$$
(15)

is the pomeron intercept and

$$E_l^{\nu}(r) = \int d^2 R \exp(ilR) \left(\frac{r}{|R + r/2||R - r/2|}\right)^{1+2i\nu}$$
 (16)

The Green function (14), transformed into momentum space, contains terms proportional to $\delta^2(l/2 \pm q)$, which should be absent in the physical solution (this circumstance was first

noted by A.H.Mueller and W.-K.Tang [9]). For that, (14) goes to zero at r = 0. Terms proportional to $\delta^2(l/2 + q)$ are not dangerous to us: they are killed by the vertex K_l . To remove the dangerous singularity at q = l/2 and simultaneously preserve a good behaviour at r = 0, as in [6], we make a subtraction in E, changing it to

$$\tilde{E}_{l}^{\nu}(r) = \int d^{2}R \exp(ilR) \left(\left(\frac{r}{|R + r/2||R - r/2|} \right)^{1 + 2i\nu} - |R + r/2|^{-1 - 2i\nu} + |R - r/2|^{-1 - 2i\nu} \right)$$
(17)

Integration over r leads to the integral

$$J_2(l,q) = \int (d^2r/(2\pi)^2)(1/r^2)\tilde{E}_l^{\nu}(r)$$
(18)

This integral is convergent at any values of ν , the point $\nu = 0$ included, when the convergence at large values of r and R is provided by the exponential factors. So in the limit $s \to \infty$, when small values of ν dominate, we can put $\nu = 0$ in J_2 :

$$J_2(l,q) = \int (d^2Rd^2r/(2\pi)^2) \frac{\exp(ilR + iqr)}{r|R + r/2||R - r/2|} + (1/l)\ln\frac{|l/2 - q|}{|l/2 + q|}$$
(19)

The second term comes from the subtraction terms in (17). Passing to the Fourier transform of the function 1/r one can represent (19) as an integral in the momentum space

$$J_2(l,q) = (1/l) \int \frac{d^2p(l+|l/2+p|-|l/2-p|)}{(2\pi)|l/2+p||l/2-p||q+p|}$$
(20)

In the same manner we can put $\nu = 0$ in the function $E_l^{-\nu}(r_{20})$ obtaining

$$\int \frac{d^2 R \exp(ilR) r_{20}}{(2\pi)^2 |R + r_{20}/2| |R - r_{20}/2|} \equiv F(l)$$
(21)

The rest of the Green function is easily calculated by the stationary point method to finally give

$$\chi_2(s, l, q) = 8\sqrt{\pi} s^{\Delta} (a \ln s)^{-3/2} F_2(l) J_2(l, q)$$
(22)

where

$$\Delta = \omega_0 = (g^2 N/\pi^2) \ln 2, \quad a = (7g^2 N/2\pi^2)\zeta(3)$$
(23)

3 The projectile part

The BFKL Green function at l=0, which enters (12) is given by the expression [8]

$$G_0(s, r, r') = (1/8)rr' \int_{-\infty}^{\infty} \frac{d\nu s^{\omega(\nu)}}{(\nu^2 + 1/4)^2} (r/r')^{-2i\nu}$$
(24)

Applying to (24) the operator ∇^4 we obtain

$$\nabla^4 G_0(s, r, r') = (2r'/r^3) \int_{-\infty}^{\infty} d\nu s^{\omega(\nu)} (r/r')^{-2i\nu}$$
 (25)

Before integrating over r in (12) we shall integrate over the energy s_1 , as indicated in (11). Part of the s_1 -dependence comes from factors χ_2 . Recall that $s_2 = s/s_1$ and that also $s >> s_1$. This latter condition allows to neglect the dependence on s_1 in the logarithmic factor in (22), so that the two factors χ_2 will only contribute a factor $s_1^{-2\Delta}$. We then obtain an integral

$$\int ds_1 s_1^{-1-2\Delta} \nabla^4 G_0(s, r, r') = (2r'/r^3) \int_{-\infty}^{\infty} d\nu (r/r')^{-2i\nu} (2\Delta - \omega(\nu))^{-1}$$
 (26)

Now we have to finally integrate over r to obtain the function χ_1 , Eq. (12), integrated over s_1 . This integration requires some care due to a high singularity of the righthand side of (26) at r=0. To do it, we consider Δ in (26) as a variable and first take $\Delta < 0$. Then the denominator in (26) will not vanish in the strip of the upper half-plane of ν with $\text{Im } \nu < \frac{1}{2} + \epsilon$, $\epsilon > 0$. This allows to shift the integration contour in (26) to a line $\text{Im } \nu = \frac{1}{2} + \epsilon$. Then the singularity in r will be diminished to make the integration over r possible. We get

$$J_1(q, r_1) \equiv \int ds_1 s_1^{-1-2\Delta} \chi_1(s_1, q, r_1) = -\pi q r_1 \int_{\text{Im } \nu = \frac{1}{2} + \epsilon} d\nu (q r_1/2)^{2i\nu} \frac{1}{2\Delta - \omega(\nu)} \frac{\Gamma(1/2 - i\nu)}{\Gamma(1/2 + i\nu)}$$
(27)

With $\Delta < 0$ the integrand on the righthand side has no singularities in the strip $\operatorname{Im} \nu < \frac{1}{2} + \epsilon$, since the pole of the Γ function at $\nu = i/2$ is compensated by the pole of the function ψ in $\omega(\nu)$ at the same point. So we can return to the integration over the real ν and subsequently pass to the physical value $\Delta > 0$. Then finally

$$J_1(q, r_1) = -\pi q r_1 \int_{-\infty}^{+\infty} d\nu (q r_1/2)^{2i\nu} \frac{1}{2\Delta - \omega(\nu)} \frac{\Gamma(1/2 - i\nu)}{\Gamma(1/2 + i\nu)}$$
(28)

This integral can be calculated as a sum of residues of the integrand at points $\nu = \pm ix_k$, $0 < x_1 < x_2 < ...$, at which

$$2\Delta - \omega(\nu) = 0 \tag{29}$$

Residues in the upper semiplane are to be taken if $qr_1/2 > 1$ and those in the lower semiplane if $qr_1/2 < 1$. Thus we obtain

$$J_1(q, r_1) = -\frac{(2\pi)^4}{g^2 N} \sum_k a_k (qr_1/2)^{1 \pm 2x_k} \frac{\Gamma(1/2 \mp x_k)}{\Gamma(1/2 \pm x_k)}$$
(30)

where

$$a_k = \frac{1}{\psi'(1/2 - x_k) - \psi'(1/2 + x_k)}$$
(31)

and the signs should be chosen to always have $(qr_1/2)^{\pm 2x_k} < 1$.

The first three roots of Eq. (29) are

$$x_1 = 0.2648, \quad x_2 = 1.3505, \quad x_3 = 2.3704$$
 (32)

with the corresponding coefficients a_k

$$a_1 = 0.05944, \quad a_2 = 0.02139, \quad a_3 = 0.01610,$$
 (33)

4 The triple pomeron cross-section

As we have found, the projectile part gives a contribution to the cross-section (12) in the form of a sum of powers $(qr_1)^{1\pm 2x_k}$, with rising values of x_k . For large Q^2 we expect that $r_1 \sim 1/Q$, so that the product qr_1 is small. Then the plus sign should be taken in (30) and the bulk of the contribution is expected to come from the lowest power x_1 . For this reason in the following we shall study the contribution from only the nearest pole $\nu = -ix_1$ to (28), taking

$$J_1(q, r_1) \simeq -\frac{(2\pi)^4}{g^2 N} a_1 (qr_1/2)^{1+2x_1} \frac{\Gamma(1/2 - x_1)}{\Gamma(1/2 + x_1)}$$
(34)

Combining all the factors, in this approximation we find for the cross-section σ , Eq. (11)

$$\sigma = \frac{g^8 N^2}{(2\pi)^6} 2^{-2x_1} a_1 B_1 \frac{\Gamma(1/2 - x_1)}{\Gamma(1/2 + x_1)} \frac{s^{2\Delta}}{(a \ln s)^3} \langle r_1^{1+2x_1} \rangle \int d^2 l \, l^{-1+2x_1} F^2(l)$$
 (35)

Here we have introduced the average value of $r_1^{1+2x_1}$ in the projectile

$$\langle r_1^{1+2x_1} \rangle = \int d^2 r_1 r_1^{1+2x_1} \rho_1(r_1) \tag{36}$$

 B_1 is a number defined as as a result of the q integration

$$B_1 = l^{1-2x_1} \int (d^2q/(2\pi)^2)|l/2 + q|^{1+2x_1} (\nabla_q J(l,q))^2$$
(37)

It does not depend on l and can be represented as an integral over three momenta

$$B_{1} = \frac{1}{(2\pi)^{4}l^{1+2x_{1}}} \int d^{2}q d^{2}p d^{2}p' \frac{|l/2+q|^{1+2x_{1}}(q+p)(q+p')(l+p_{+}-p_{-})(l+p'_{+}-p'_{-})}{p_{+}p_{-}p'_{+}p'_{-}|q+p|^{3}|q+p'|^{3}}$$
(38)

where

$$p_{\pm} = |p \pm l/2|; \quad p'_{\pm} = |p' \pm l/2|$$
 (39)

It is a well-defined integral. B_1 generalizes a similar constant B which appears in the asymptotic triple pomeron vertex [6], in which x_1 is absent. Calculations give

$$B_1 = 6.84$$

The explicit form of F(l) is given by (21). So its square introduces two more integrations, over R and R'. Integration over l then gives

$$\int d^2l \, l^{-1+2x_1} \exp il(R - R') = 2^{1+2x_1} \pi \frac{\Gamma(1/2 + x_1)}{\Gamma(1/2 - x_1)} |R - R'|^{-1-2x_1} \tag{40}$$

We are left with the final integral over R and R'

$$\int \frac{d^2Rd^2R'}{(2\pi)^4|R-R'|^{1+2x_1}} \frac{1}{|R+r_{20}/2||R-r_{20}/2||R'+r_{20}/2||R'-r_{20}/2|} = \frac{D}{r_{20}^{1+2x_1}}$$
(41)

which defines another numerical constant D. Numerical integration gives

$$D = 0.566$$

Putting this into (35) we obtain the final cross-section

$$\sigma = \frac{g^8 N^2}{(2\pi)^5} a_1 B_1 D \frac{s^{2\Delta}}{(a \ln s)^3} r_{20}^{1-2x_1} \langle r_1^{1+2x_1} \rangle \tag{42}$$

with the known numerical constants a_1 , B_1 and D.

The Q^2 -dependence of the cross-section (42) is concentrated in the average value $\langle r_1^{1+2x_1} \rangle$. At large Q^2 this average has the order Q^{-1-2x_1} , which determines the order of the cross-section σ to be

$$\sigma \sim (r_{20}/Q)(Qr_{20})^{-2x_1} \tag{43}$$

This should be compared with the cross-section which results from the pure two-pomeron exchange, Fig. 1b. As found in [1], it has the same dependence on s but falls only as 1/Q at large Q^2 . Therefore for large Q^2 the triple pomeron contribution is neglegible relative to the pure two-pomeron exchange, due to the reduction of the anomalous dimension by $2x_1$. This is the main result of our study.

Two comments are to be added in conclusion. First, one might think that the obtained result is only a consequence of different scales of the projectile and target. Taking the virtality of the target of the same order Q^2 , one might argue that $\sigma \sim 1/Q^2$ on dimensional grounds, which is of the same order as for the pure two pomeron exchange. However this argument would be wrong. With a highly virtual target, one cannot simply put $r'_2 = r'_3 = r_{20}$, but has to consider the perturbative density ρ_2 , which is singular at the origin. Then one has to regularize the integrations in r'_2 and r'_3 by introducing a finite mass m for quarks inside the target. As a result, the average values of $r'_{2,3}$ will not have the order 1/Q but rather 1/m. Then the final conclusion will remain the same: the contribution will be damped by the factor $(m/Q)^{2x_1}$.

Second, the procedure followed here for the nearest pole at $\nu = -ix_1$ cannot be trivially generalized to other poles. The point is that with $x_k > 1/2$ neither the integral (38) nor the integral (41) converge. For such x_k one has to use the general form (30), taking different signs in different parts of the (q, r_1) phase space. Then the integrals over r_1 , q, R and R' do not decouple and the cross-section turns out to be represented by a very complicated 12-dimensional integral over r_1 , q, R, R', p and p'. However one can estimate the resulting Q^2 dependence by noting that for $x_k > 1/2$ the extra dimension in (38) will be supplied by the corresponding power of Q. One then finds that for all $x_k > 1/2$ the cross-section has the same order $1/Q^2$. Thus, although the contributions of all $x_k > 1/2$ are definitely smaller

than the one from x_1 , they all have to be calculated simultaneously. Therefore our derivation is only practical for the dominant contribution corresponding to the pole at $\nu = -ix_1$.

5 Conclusions.

We have shown that for highly virtual probes the triple (and hopefully multiple) pomeron interaction does not contribute to the cross-section, so that it can be calculated as a sum of independent multipomeron exchanges, as has been done in [1]. The amplitude then aquires an essentially eikonal form.

For colliding low-mass particles the triple pomeron does contribute significantly. We have not been able to find this contribution in a form suitable for practical calculations. However we would like to stress that even if we had succeded, that would not have solved the problem. On the one hand, when a higher number of pomerons is exchanged higher order multipomeron interactions evidently come into play, whose calculation is still more hopeless. On the other hand, for low-mass particles the coupling to a pomeron (or to several pomerons) is essentially non-perturbative. So even without any multipomeron interactions calculation of multipomeron exchanges becomes hardly possible. Multipomeron interactions then do not make the situation significantly worse, only adding a contribution which can be calculated perurbatively, in principle, but not in practice.

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7 References

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8 Figure captions

- 1. Triple pomeron (a) and "pure" double pomeron exchange (b) contributions to the scattering amplitude.
- 2. The generic double pomeron exchange diagram for the scattering amplitude.

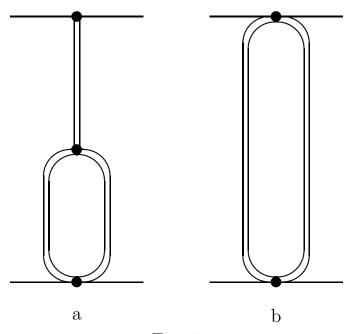


Fig. 1

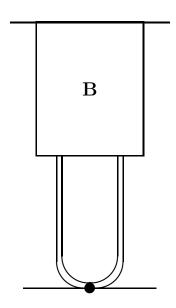


Fig. 2